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Manuscript received October 21, 1964; revision received September 10, 1965; paper accepted September 13, 1965.

Forced Convection Mass Transfer:

Part III. Increased Mass Transfer from a Flat Plate Caused by the Wake from Cylinders Located Near the Edge of the Boundary Layer

DAVID G. THOMAS

Oak Ridge National Laboratory, Oak Ridge, Tennessee

Enhanced rates of mass transfer in the wake region behind detached cylindrical turbulence promoters were investigated with the use of the naphthalene sublimation technique. The maximum increase in the average rate of mass transfer through laminar boundary layers was over 170%. The remarkable feature of these results was that the enhanced rates of mass transfer persisted for over one hundred and thirty cylinder diameters downstream from the cylinder generating the wake. The observed effect was not only strongly dependent on the free stream velocity and the location of the cylinders relative to the mass transfer surface, but there were marked differences observed between the rate of mass transfer in the wake region behind one and behind two cylinders. These results resemble a "tuning phenomenon" and are believed to be due to Tollmein-Schlichting instabilities and premature transition.

In the second paper of this series (*1*) it was shown that the local and average rates of forced convection through laminar boundary layers on a flat plate were markedly increased by locating small cylinders near the outer edge of the boundary layer. The local rate of forced convection was strongly peaked directly beneath each cylinder; the magnitude of the effect depended upon the free stream velocity, the spacing between cylinders, and the gap between the cylinders and the plate. Under optimum conditions, local values of the rate of forced convection were increased as much as 240%, while the average values were increased by over 90%. The principal factors responsible for the observed peaking of the local rate of forced convection beneath cylinders were believed to be an increase in the velocity gradient at the surface and the interaction of the cylinder wake with the fluid in the boundary layer (*1*).

The wake interaction is undoubtedly quite complicated. In the first place, for the cylinder diameter and velocities covered in the second paper (*1*), the Reynolds number of the cylinder was above the threshold value for vortex street formation (*2*), provided the cylinder was located far from a surface. In other studies (*3*) it was shown that when multiple cylinders were located one behind the other in the plane of flow of the free stream, the vortex streets interacted in a very complex fashion. Contraction, expansion, cancellation, and coalescence of vortices were observed to occur for different values of cylinder separation and Reynolds number. Little information is available on the further complication introduced because of the proximity of the cylinders to a surface.

In addition to its intrinsic interest the isolation of wake interaction effects on the local rate of forced convection is a prerequisite to the understanding of the complicated dependence of the magnitude of the increase in the local rate of forced convection beneath cylinders on the velocity and/or the length Reynolds number (*1*).

WAKE INTERACTIONS

Many instances of interactions of wakes and secondary flows with fluid in the boundary layer are known. In general, the result of the interactions appears as excitations of inherent instabilities [such as Tollmein-Schlichting waves (*5*)], as secondary flows, or as other modifications of the boundary-layer velocity profile which may cause an increase in the rate of forced convection. However, little information is available on the magnitude of the increase in the rate of forced convection which would be caused by these flow modifications.

Tollmein-Schlichting waves and eventually premature boundary-layer transition have been shown to be excited by four or six 0.006-in. diameter wires located a few inches apart attached to the surface of a convex plate with a 20-ft. radius of curvature (*6*) and by multiple wires 0.002- to 0.1875-in. diameter in contact with the surface of a flat plate (*7, 8*). Hot wire investigations showed (*6*) that maximum amplification of the Tollmein-Schlichting waves and transition did not necessarily occur directly at the location of the cylinders, but could occur some distance downstream. In addition, premature transition caused by the cylinders occurred either in the cylinder wake or further downstream after the boundary layer returned to the normal Blasius type of profile.

In the studies cited above (*6* to *8*), the flow disturbances were generated by cylinders located directly on the surface. Similar effects have been observed when the turbulence generator was detached from the surface. For instance when a lateral variation in boundary-layer thickness was produced by a series of streamwise tip vortices shed from five small wings located one beside the other parallel to a flat plate, normal to the direction of the flow and outside the boundary layer, Tollmein-Schlichting waves rapidly developed nonlinear characteristics leading directly to the onset of turbulence (*9*). The development of the nonlinear flow characteristics required a certain Tollmein-Schlichting wave intensity which was a function of the three-dimensionality of the boundary layer (*9*).

Vortices, generated as a secondary flow from a variety of geometries (*10* to *13*), have been shown to increase the rate of forced convection particularly under laminar flow conditions. In studies of the mechanism of finned tube heat transfer, a 25% increase in the rate of laminar forced convection from a flat plate was obtained when the plate was placed in the wake of an identical unheated flat plate operating in the range of stable vortex shedding (*10*). Vortices shed from rotating triangular plates, positioned with their apexes about half way between the center and side wall of a cylindrical vessel, proved to be an efficient means for increasing the rate of forced convection to a cylindrical surface (*11*). The conditions for optimum vortex production from the sharp swept-back edges of the triangular plates were determined by flow visualization studies (*11*). Principles determined in the above study (*11*) were used to design vortex generators for use in a refrigerating duct, resulting in a marked improvement in overall plant efficiency (*12*).

The mechanics of the formation of Taylor vortices, which form when fluids rotating in the gap between concentric cylinders possess a Taylor number in excess of a certain critical value, have been studied both theoretically and experimentally (*13, 14*). In forced convection studies, the secondary flow of the Taylor vortices has been observed to cause as much as a 200% increase in the Nusselt number compared to the values obtained below the critical Taylor number (*15, 16*).

To summarize, the wake from cylinders placed directly on a flat or slightly convex surface excites Tollmein-Schlichting waves and promotes early transition. Neither event necessarily occurs at the location of the cylinders but could occur some distance downstream even after the velocity profile returned to its normal shape. No information is known on the effects of such phenomena on the rate of forced convection. Secondary flows [that is, Taylor vortices (*15, 16*), Wing tip vortices (*11*), or vortex streets from flat plates (*10*)] increased the average rate of forced convection through laminar boundary layers by from 25 to 200%, depending on the method of formation.

The object of this investigation was to determine the magnitude and extent of increases in the rate of forced convection from a flat plate in the wake region of multiple cylinders detached from the surface and located near the edge of the laminar boundary layer. A second objective was to determine whether there were optimum combinations of fluid velocity and cylinder location which caused maxima in the rate of forced convection in the wake region. This possibility was suggested by flow visualization studies of vortex street interactions in which expansion, contraction, and beats were observed in the vortex street wake when two cylinders were placed one behind the other at different spacings (*3*).

EQUIPMENT AND PROCEDURE

The local rate of forced convection through laminar boundary layers on a flat plate was determined by measuring the difference in level of a flat naphthalene plate before and after exposure to an air stream. The experiments were carried out in a once-through wind tunnel with a 6-ft. long 3 × 12-in. working section made from 1-in. thick transparent plastic.

The naphthalene was contained in a 1/16-in. deep by 6 × 17-in. recess in a 11 15/16 × 18 × 3/16-in. flat stainless steel plate. The leading edge of the plate was sharpened to a knife edge flat on the upper side with 11 deg included angle; the plate was supported by four strips of metal 16½-in. long, 7/32-in. wide, and ½-in. high. The surface level of the naphthalene was measured to the nearest 0.0001 in.; it is estimated (*21*) that less than 0.00005 in. of naphthalene sublimed during the surface measurement. The duration of a test run was chosen so that at least 0.002 and less than 0.015 in. of naphthalene sublimed. All runs were made at ambient temperature to minimize temperature variations along the naphthalene surface. The temperature depression of the mass transfer surface due to sublimation of the naphthalene was estimated (*21*) to be always less than 0.2°C. Flow velocities were measured with a pitot tube, a calibrated vane type of anemometer, or with smoke tracers, depending on the velocity range used for a particular test.

The mass transfer Stanton number corresponding to the difference in surface level of the naphthalene caused by exposure of the flat plate in the wind tunnel was calculated from the expression:

$$N_{St} = \frac{k_o}{U_o} = \frac{N_w \, P_{bar} \, M_A}{P_N \, \rho_A \, U_o \, M_N} \tag{1}$$

and the Schmidt number for the naphthalene-air system ($N_{Sc} = 2.44$ at 25°C.) was calculated from the expression:

$$N_{Sc} = 7(T_K)^{-0.185} \tag{2}$$

recommended by Sherwood and Träss (*21*). Additional details on the equipment and procedure may be found elsewhere (*4*).

The detached turbulence promoters used in this study were hollow cylinders 0.093-in. OD and 0.070-in. ID and 11¾-in. long. Hollow cylinders were used to minimize the sag of the cylinders under their own weight. The cylinders were supported by special holders resting on the side edges of the stainless steel plate. The centerline of the cylinders could be located at distances of 1.6, 2.2, and 3.2 cylinder diameters from the surface of the plate (gaps between the bottom of the cylinder and the plate of 1.1, 1.7, and 2.7 cylinder diameters). Multiple cylinders could be located one behind the other at uniform center-to-center distances of 6.5, 8.9, 10.8, 16.1, and 32.3 diameters.

The first detached cylindrical turbulence promoter was always placed at least 2 in. downstream from the leading edge. Therefore, a portion of the data from each run provided an internal calibration which gave the value of the rate of mass transfer in the absence of turbulence promoters for the particular conditions being studied. This permitted a more accurate evaluation of the maximum local value of the mass transfer coefficient and of the integrated average values achieved with any given promoter.

EXPERIMENTAL RESULTS

The extent and magnitude of the increased mass transfer that can be achieved in the wake region with different combinations of cylinder spacing one behind the other are illustrated in Figure 1. In this figure the dimensionless rate of mass transfer k_c/U_o is plotted against both distance from the leading edge and the length Reynolds number for a plate with no turbulence promoters, for a plate with a single cylinder located with its center 2.1 diameters from the surface, for two cylinders located with

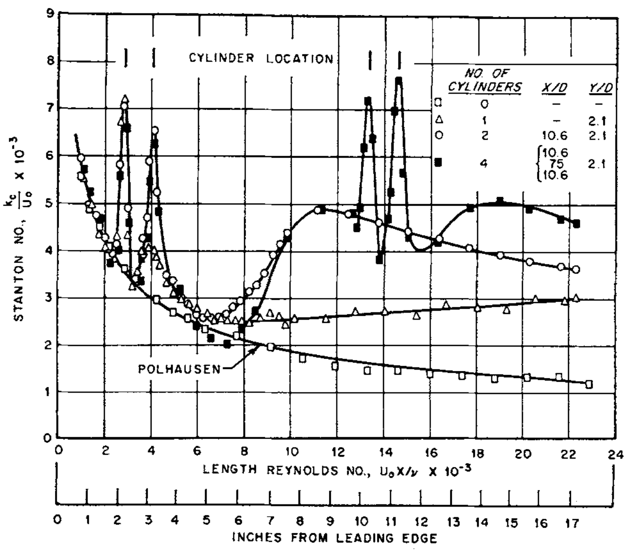

Fig. 1. Effect of placing one, two, or four 0.093 in. diameter cylinders near the edge of the laminar boundary layer on the value of the local mass transfer Stanton number (Schmidt number = 2.44).

centers 2.1 diameters from the surface and separated by a center-to-center distance of 10.6 diameters, and for four cylinders located as pairs with center-to-center separation of 2.1 diameters with the second pair separated by seventy-five cylinder diameters from the first pair. The free stream velocity was 2.72 ft./sec. for all the data shown in Figure 1.

The data obtained with no cylinders were in good agreement with the theoretical curve derived by Polhausen (*22*) over the entire length of the plate with no evidence of transition to a turbulent boundary layer. [Previous studies (*4*) showed that for the turbulence levels existing in the present wind tunnel, transition in the absence of cylinders normally occurred at length Reynolds numbers greater than 7×10^4 for all the conditions used in the present study; for the particular conditions of Figure 1, transition in the absence of cylinders should occur at a length Reynolds number of 7×10^5.]

The data obtained with a single detached cylindrical turbulence promoter showed a pronounced maximum directly beneath the cylinder. The sharp, narrow maximum always occurred directly beneath the cylinder and serves as a ready reference to determine the cylinder location. Since the characteristics of this local maximum have been described in detail elsewhere (*1*), no further comments will be made about these peaks, except in those instances when the present study discloses a new aspect of their behavior. A small increase in the rate of forced convection was observed in the wake region behind the single cylinder with the effect increasing slowly with distance from the cylinder. However, when two cylinders were located one behind the other, there was a marked increase in the rate of forced convection in the wake region. The pronounced difference in the rate of forced convection caused by the second cylinder began 5 in. from the leading edge of the plate (~ 2 in. downstream of the second cylinder) and continued to a distance of 17½ in. from the leading edge, the end of the mass transfer surface. The location of an additional two cylinders in the wake region seventy-five cylinder diameters downstream from the first two resulted in two pronounced peaks directly beneath these cylinders as well as a further increase in the wake of the third and fourth cylinders. As can be seen from Figure 1, the effect of the second pair of cylinders is directly additive upon the effect of the first pair.

The increase in the average rate of mass transfer in the wake region caused by two cylinders is 144% compared to only 61% increase observed in the wake from a single cylinder. With four cylinders located at the positions shown in Figure 1, the increase in the average rate of mass transfer over the last 12½ in. of the plate (the wake region of the first pair of cylinders) was 175%.

As yet there are no data available on the increase in pressure drop caused by the location of cylinders near the edge of the boundary layer; however, data from the literature (*17*) show that the drag of a cylinder located ten diameters behind another far from all other surfaces is only 30% of the drag of the leading one. If this proves to be even approximately true when the cylinders are located close to a surface, then the more than doubling of the wake effect caused by the proper positioning of two cylinders would be accompanied by only a modest increase in the pressure drop of the single cylinder system.

The results illustrated in Figure 1 resemble a "tuning phenomenon" in that they are functions of the separation of the cylinders, the distance of the cylinders from the plate, and the velocity of the air stream. The complicated behavior for single cylinders and for two cylinders separated by a center-to-center distance of 10.6 diameters is shown as a function of distance from the plate and velocity in Figures 2 and 3. These figures are a plot of local Stanton number vs. length Reynolds number; note that the first point for all the different sets of data is always in the Reynolds number range $10^3 < (N_{Re})_x < 10^4$. For comparison, curves for laminar and turbulent flow in the absence of cylinders are shown for each set of data; these curves are labeled only in Figures 2*A* and 3*A*; however, they are duplicated for all the other sets of data. The velocity and location of the cylinders for each set of data are given in Table 1; the location of the cylinders in Figures 2 and 3 can be identified readily by the sharp peaks at the left of each set of data. The data to the right of these peaks will be referred to as being in the wake region.

The local rate of mass transfer in the wake region proved to be very complex with both pronounced maxima and minima. In a few cases, two maxima were observed in addition to those associated with the region beneath the cylinders. For instance, in Figure 2*B*, the maximum beneath the cylinder occurred at $(N_{Re})_x = 4.4 \times 10^3$ and two other maxima were observed at $(N_{Re})_x = 6.0 \times 10^3$ and 14.0×10^3. When the cylinder was moved farther from the surface (from $Y/D = 1.6$ to 2.2, Figures 2*B* and 2*E*, respectively), the maximum at $(N_{Re})_x = 14.0 \times 10^3$ disappeared, but the maximum at $(N_{Re})_x = 6.0 \times 10^3$ markedly increased. Comparison of Figures 2*D*, 2*E*, and 2*F* showed that for a single cylinder whose center was 2.2 diameters above the plate, the value of the local Stanton number at the wake maximum went through a maximum for a velocity of 4.4 ft./sec., Figure 2*E*. Finally, Figures 2*G* and 2*I* are typical examples of data in which a minimum was observed in the local Stanton number–length Reynolds number curve in the wake region.

Turning now to Figure 3, which shows data obtained under comparable conditions with those of Figure 2, except that two cylinders were placed one behind the other with a center-to-center spacing of 10.6 diameters, one

TABLE 1. MEAN STREAM VELOCITY AND DISTANCE OF CYLINDERS FROM SURFACE FOR DATA OF FIGURES 2 AND 3

	U_o, ft./sec.		
Y/D	3.5	4.4	6.2
1.6	A	B	C
2.2	D	E	F
3.2	G	H	I

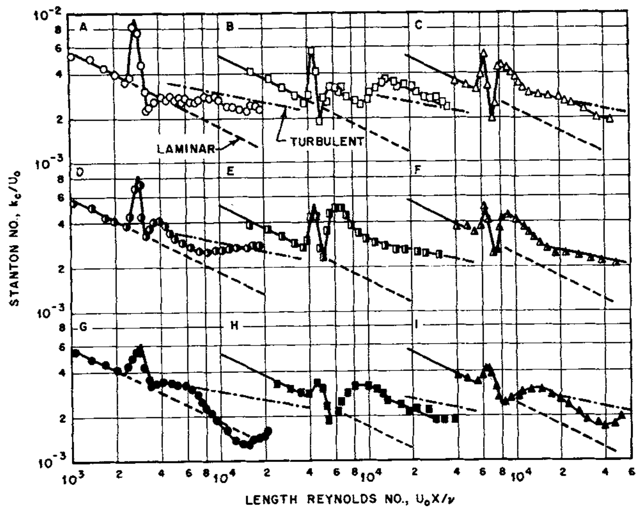

Fig. 2. Effect of a single 0.093 in. diameter cylinder located near the edge of the laminar boundary layer on the rate of mass transfer from a flat plate (Schmidt number = 2.44).

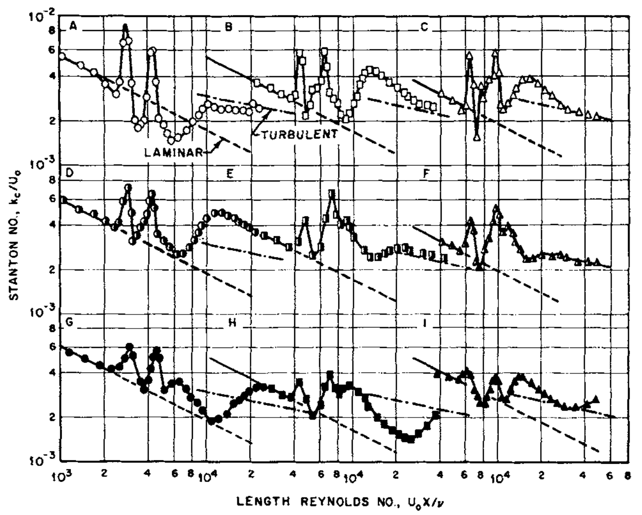

Fig. 3. Effect of two 0.093 in. diameter cylinders located near the edge of the laminar boundary layer on the rate of mass transfer from a flat plate (Schmidt number = 2.44).

observes many similarities and some differences. For instance, in Figure 3*B*, the second cylinder was located near the first wake maximum from the first cylinder ($U_o x/\nu$ = 6.4 × 10^3 and 6.0 × 10^3, respectively) with the result that the second cylinder mass transfer peak was proportionally larger than the first cylinder mass transfer peak based on the rate of mass transfer in the absence of cylinders. There was also a wake maximum in the rate of mass transfer in Figure 3*B* at the same location as the second wake maximum of Figure 2*B* ($U_o x/\nu = 1.4 \times 10^4$); however, the magnitude of the peak was somewhat greater than in Figure 2*B*.

In Figures 3*D*, 3*E*, and 3*F*, the second cylinder was located near the wake maximum from the first cylinder, and the centers of both cylinders were 2.2 diameters from the surface. In this case the largest value of the rate of mass transfer beneath the second cylinder was observed at $U_o x/\nu = 7.2 \times 10^3$ in Figure 3*E*. (Note that in the series 2*D*, 2*E*, and 2*F*, the largest wake maximum was also observed at this same length Reynolds number in Figure 2*E*). However, the largest wake maximum in the series 3*D*, 3*E*, and 3*F* was observed in Figure 3*D* at $U_o x/\nu = 1.1 \times 10^4$ (compare with Figure 2*D* in which no maximum was observed at this Reynolds number).

For the data shown in Figures 3*G*, 3*H*, and 3*I*, the cylinder centers were located 3.2 diameters from the surface. Comparison of these three sets of data with their counterparts in Figure 2 shows that the maximum in the rate of mass transfer due to the wake of the first cylinder was substantially unaffected by the second cylinder and, further, that either there was no wake maximum from the second cylinder or it occurred so far downstream that it was seldom observable with the length of test plate used in these studies.

Wake Maximum Location

The length Reynolds number at the maximum in the rate of forced convection in the wake region is plotted in Figure 4 vs. the length Reynolds number corresponding to the location of the downstream cylinder (that is, the location of the single cylinder or the location of the second of two cylinders). For single cylinders, the location of the maximum in rate of mass transfer in the wake region was substantially a constant factor times the location of the cylinder, that is

$$\frac{[(N_{Re})_x]_{\text{wake maximum}}}{[(N_{Re})_x]_{\text{cylinder location}}} = A \qquad (3)$$

where *A* had values of 1.3, 1.45, and 2.2 for cylinders with a spacing beneath the bottom of the cylinder and the mass transfer surface of Y/D = 1.6, 2.2, and 3.2. Since the single cylinder was always located 2 in. from the leading edge, this means that for any given value of Y/D, the wake maximum occurred at a constant distance behind a single cylinder, independent of velocity. The distance of the wake maximum behind the cylinder in terms of the constant *A* is

$$\text{Distance} = x_{WM} - x_{CL} = (A - 1)x_{CL} \qquad (4)$$

This gives distances of 0.6, 0.90, and 2.4 in. for Y/D of 1.6, 2.2, and 3.2. In a few experiments with Y/D of 4.3, the wake maximum occurred far downstream at a distance of 10 in. from the single cylinder.

However, the distance of the wake maximum behind the second of two cylinders was a rather complicated function of the velocity. For instance, when $Y/D = 2.2$, the distance of the wake maximum increased from 3.9 to 5.5 in. as the length Reynolds number increased from 1.3 × 10^3 to 7 × 10^3. The distance then decreased with increasing Reynolds number in the range $7 \times 10^3 < (N_{Re})_x < 1.3 \times 10^4$ from 5.5 to 0.87 in. for $Y/D = 2.2$. For length Reynolds numbers greater than 1.3 × 10^4, the wake maximum occurred at a constant distance behind the second cylinder, independent of velocity, 0.45 and 0.87 in. for Y/D = 1.6 and 2.2.

It is well known that the spread of a turbulent wake (*23*) from a cylinder is proportional to $x^{1/2}/U_o$, while the spread of a laminar wake (*24*) is proportional to $(x/U_o)^{1/2}$. However, in the vortex street range of Reynolds numbers, the spread of the wake (that is, the spacing of the vortices) is substantially independent of the velocity and distance downstream after the initial expansion and is primarily a function of cylinder diameter (*2*). The rapid expansion near the cylinder is virtually completed within the first twenty cylinder diameters downstream (*25*).

The difference in behavior of the location of the wake maximum with one and with two cylinders is in general agreement with the differences in behavior of the vortex streets from one (*2*) and two (*3*) cylinders. The location of the wake maximum in the region behind a single cylinder was virtually independent of velocity and the strong dependence on Y/D was also consistent with a rapid expansion of the wake within the first twenty diameters downstream and rather slow expansion at further distances downstream. With two cylinders, the location of

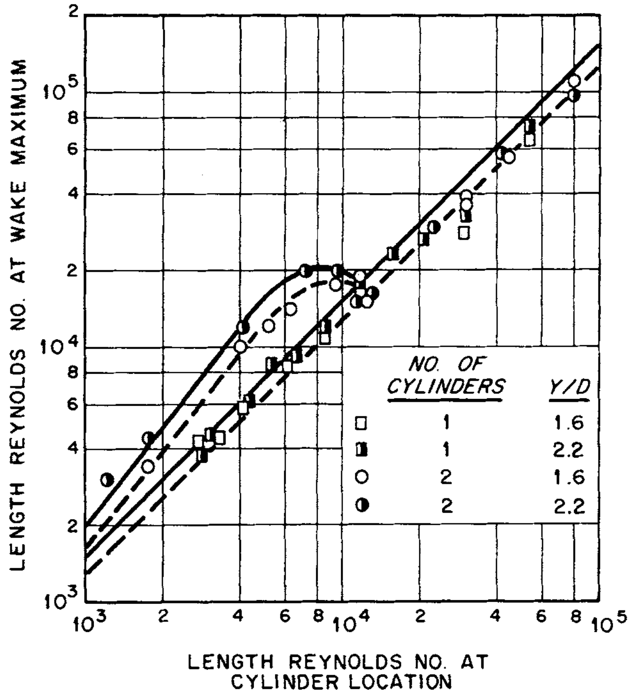

Fig. 4. Location of maximum in rate of mass transfer in the wake region in relation to the location of the cylinder generating the wake.

the wake maximum was a function of velocity in the Reynolds number range in which expansion and contraction of the vortex street was observed in flow visualization studies (3). In addition, the flow visualization studies indicated that these phenomena began at a considerable distance downstream from the cylinder (greater than twenty-five diameters) in accord with the observation that the wake maximum behind two cylinders occurred at distances almost a factor of three to five greater than from a single cylinder.

Wake Enhancement of Forced Convection

The maximum increase in the value of the local rate of mass transfer in the cylinder wake region was a factor of three. This was observed (Figure 3*B*) when two cylinders were placed one behind the other 10.6 diameters apart with their centers 1.6 diameters from the surface and with a free stream velocity of 4.4 ft./sec. Although the peak values of the local mass transfer Stanton number are of interest, the significant values are the amount the average Stanton number was increased by the detached turbulence promoters. Consequently, the ratio

$$\frac{[(N_{St})_{\text{cylinder}}]_{\text{average}}}{(N_{St})_{\text{average}}} = \frac{\int_l^L (N_{St})_{\text{cylinder}}\, dx}{\int_l^L (N_{St})\, dx} \tag{5}$$

was determined for two different cases: (A) the combined effect of the cylinders and the wake for which the lower limit of the integral was taken as $l = 16D$ ($5D$ upstream from the first cylinder) to $L = 183D$ (the end of the mass transfer surface), and (B) the effect of the wake alone for which the lower limit of the integral was taken as $l = 51D$ ($5D$ downstream of the second cylinder) to $L = 183D$. The values of this ratio [Equation (5)] for cases (A) and (B) are shown in Figure 5 as a function of length Reynolds number, LU_o/ν, with the distance of the cylinders from the flat plate as a parameter.

Two distinctive features of the increased average rate of mass transfer in the wake region which Figure 5 illustrates are the broad maximum observed with all three cylinder locations in the range $3 \times 10^4 < LU_o/\nu < 6 \times 10^4$ and the rather narrow maximum observed for $Y/D = 2.2$ and 3.2 in the range $2.0 \times 10^4 < LU_o/\nu < 2.7 \times 10^4$. Both maxima had substantially the same value, $\{[(N_{St})_{\text{average}}]_{\text{cylinder}}/(N_{St})_{\text{average}}\} \approx 2.25$ for case *A* and 2.5 for case *B*. The broad maxima in Figures 5*A* and 5*B* occurred for "length Reynolds numbers at cylinder location" corresponding to the transition region shown in Figure 4. In this transition region the location of the wake maximum moved to positions relatively closer to the downstream cylinder as the length Reynolds number at the cylinder location was increased.

Not only does the distance of the cylinders from the plate have a strong effect on the increase of the average mass transfer Stanton number, but the separation between cylinders also has a very marked effect. The effect of cylinder spacing on the increase of the average mass transfer Stanton number is shown in Figure 6 in which the value of Equation (5) at $LU_o/\nu = 4 \times 10^4$ is plotted vs. X/D with Y/D as a parameter. The maximum increase observed in the average mass transfer Stanton number was obtained for two cylinders with centers spaced 8.9 diameters apart and located with their centers 1.6 diameters from the surface. As the spacing between cylinders was increased to distances greater than 11.6 diameters, there was a relatively slow increase in the average mass transfer Stanton number. It is believed that this is largely due to premature transition of the laminar boundary layer as the second cylinder was moved toward the downstream end of the plate.

DISCUSSION

The studies with detached turbulence promoters clearly show that a disproportionate increase in the local and average rate of mass transfer (in the air-naphthalene system) is observed in the wake region when two cylinders, located one behind the other, are positioned at certain specific separations between the cylinders and distances from the mass transfer surface. The observed effect is not

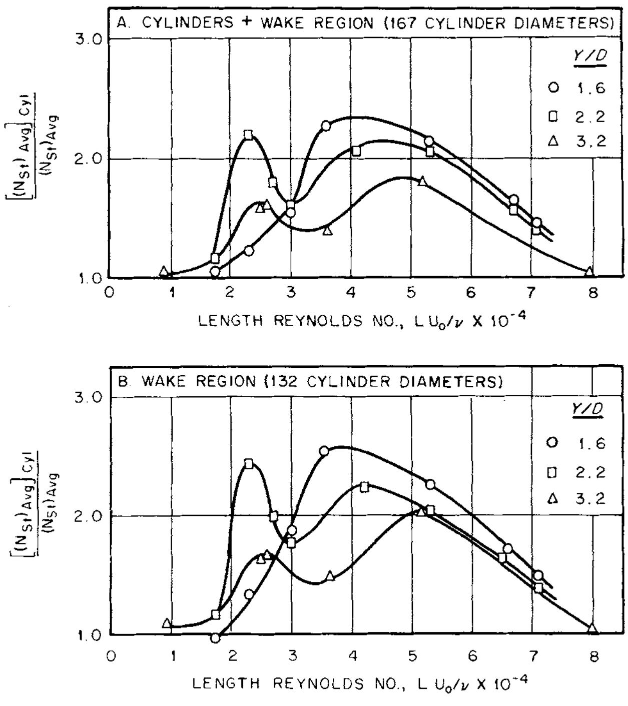

Fig. 5. Effect of two 0.093 in. diameter cylinders spaced 10.6 diameters apart near the edge of the laminar boundary layer on the average Stanton number (Schmidt number = 2.44).

only strongly dependent on the cylinder positions, but there are marked differences observed between the rate of mass transfer in the wake region behind one and behind two cylinders.

The remarkable features of these data exploring the effect of detached turbulence promoters on the increased value of the local mass transfer Stanton number for laminar boundary layers are:

1. The marked difference between the rate of forced convection in the wake region behind one and behind two cylinders, Figure 1.
2. The persistence of the increased rate of mass transfer in the wake region for over one hundred and thirty cylinder diameters downstream.
3. The apparent additivity of the effect directly beneath the cylinder and the effect in the wake region, Figures 1, 2, and 3.
4. The difference in the behavior of the location of the wake maximum behind one and behind two cylinders, Figure 4.
5. The double peak in the curves of $[(N_{St})_{\text{average}}]_{\text{cylinder}}/(N_{St})_{\text{average}}$ vs. LU_o/ν, Figure 5.
6. The dependence of the value $[(N_{St})_{\text{average}}]_{\text{cylinder}}/(N_{St})_{\text{average}}$ on the spacing between cylinders with a maximum at $X/D = 8.9$, Figure 6.

Although the mechanisms responsible for the increased rate of forced convection have not been identified, the picture that appears to be emerging is that the peak which occurs directly beneath the cylinder (*1*) represents the combined effect of the increased shear in the fluid at the surface caused by the flow around the cylinders and the effect of the wake from any upstream cylinders. The increased rate of mass transfer in the wake region (the "wake effect") is believed to be a manifestation of the sensitivity of the flow near the wall to disturbances in the free stream at or near the outer edge of the boundary layer (*18* to *20*). It has been shown theoretically (*20*) that the bulk of the effect due to the disturbances is concentrated near the wall and that the result is a greater shear stress at the wall for the disturbed profile when compared with the undisturbed profile (*18*).

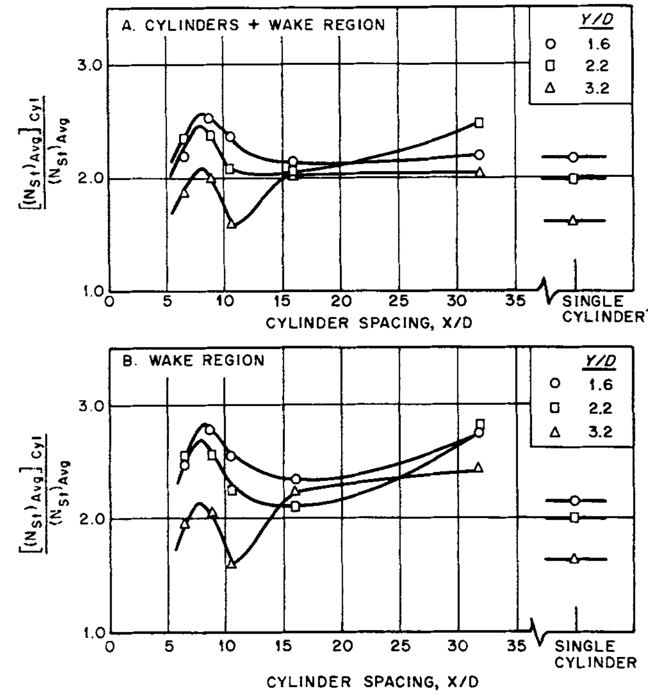

Fig. 6. Effect of cylinder location on the average Stanton number at a length Reynolds number LU_o/ν of 10^4 (Schmidt number = 2.44).

The amplification of a disturbance in a laminar boundary layer is a function of both the initial amplitude and frequency of the disturbance (5). In the particular case of Tollmein-Schlichting waves, the maximum amplification factor is associated with lower frequency disturbances (5). The present results are consistent with this observation, since the peak in the ratio $[(N_{St})_{\text{average}}]_{\text{cylinder}}/(N_{St})_{\text{average}}$ (Figure 6) occurred at a cylinder spacing of 8.9 X/D, a spacing which previous studies (3) have shown to be responsible for very complex wake interactions, with both contraction and expansion of the wake being observed. These "beat phenomena" provide the possibility of a low-frequency disturbance which could lead to high amplification Tollmein-Schlichting waves. These waves could then be the factor responsible for the peak in Figure 5 at $LU_o/\nu = 2.5 \times 10^4$ with the broad peak at $LU_o/\nu = 4.0 \times 10^4$ being due to premature transition. However, extensive measurements are required to confirm the postulated mechanism.

ACKNOWLEDGMENT

The research upon which this paper was based was sponsored by the Office of Saline Water, U. S. Department of the Interior under Union Carbide Corporation's contract with the U. S. Atomic Energy Commission.

The author wishes to acknowledge the support and suggestions of Kurt A. Kraus and the assistance of P. H. Hayes in performing the experimental measurements.

NOTATION

A = numerical coefficient, Equation (3), dimensionless
D = cylinder diameter, ft.
$\mathfrak{D}$ = molecular diffusivity, sq. ft./sec.
k_c = mass transfer coefficient, ft./sec.
l = lower limit of integral, Equation (5), ft.
L = length of mass transfer surface, ft.
M_A = molecular weight of air, lb.$_m$/mole
M_N = molecular weight of naphthalene, lb.$_m$/mole
$(N_{Re})_L$ = total length Reynolds number, LU_o/ν, dimensionless
$(N_{Re})_x$ = local length Reynolds number, xU_o/ν, dimensionless
N_{Sc} = Schmidt number, $\nu/\mathfrak{D}$, dimensionless
N_{St} = mass transfer Stanton number, k_c/U_o, dimensionless
$(N_{St})_{cyl}$ = mass transfer Stanton number in the presence of detached cylinders, dimensionless
N_w = mass flux, lb.$_m$/(sq. ft.) (sec.)
P_{bar} = barometric pressure, atm.
P_N = naphthalene vapor pressure, atm.
T_K = temperature, °K.
U_o = free stream velocity, ft./sec.
x = distance from leading edge of plate, ft.
x_{CL} = distance from leading edge of plate to cylinder location, ft.
x_{WM} = distance from leading edge of plate to wake maximum location, ft.
X = center-to-center distance between cylinders, ft.
Y = distance from cylinder center to flat plate, ft.
ν = kinematic viscosity, ft./sec.
ρ_A = density of air, lb.$_m$/cu. ft.

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Manuscript received July 23, 1965; revision received September 27, 1965; paper accepted September 29, 1965.

Tracking Function Approach to Practical Stability and Ultimate Boundedness

W. O. PARADIS
University of Illinois, Urbana, Illinois
D. D. PERLMUTTER
University of Pennsylvania, Philadelphia, Pennsylvania

A graphical method of analysis is presented for studying the practical stability and ultimate boundedness of autonomous second-order systems. It is argued that these measures of stability are in many cases more germane to design than Liapunov stability. The method incorporates much of the geometric character of a Liapunov analysis, but it is shown that a Liapunov function, relatively difficult to obtain, can be replaced by a set of easily postulated scalar functions which collectively yield the required stability information. Examples are given which demonstrate the use and effectiveness of the method.

LaSalle and Lefschetz (*7*) have made a distinction between two kinds of stability that has important implications for engineering analysis. They consider *practical* stability as distinct from stability according to the definition of Liapunov. The distinction can be put in terms of mathematical δ, ϵ statements. For autonomous systems, Liapunov stability requires that there be a $\delta(\epsilon)$ for arbitrary $\epsilon > 0$, such that a trajectory starting in $||\mathbf{x}_0|| < \delta$ always be confined within $||\Phi|| < \epsilon$. In contradistinction practical stability requires only that *some* δ, ϵ combination exist of practical size; the quantities δ and ϵ correspond to regions in x space which are chosen a priori as the ranges of expected disturbances and allowable deviations, respectively. A necessary consequence of the definition is that the δ region be a subject of the ϵ region.

It is true that in many cases a system can be stable in both senses. On the other hand, one does not imply the other as can be seen from some simple counterexamples. Consider, for example, a phase-plane portrait such as Figure 1, which might represent the behavior of a stirred reactor similar perhaps to those studied by Aris and Amundson (*1*). A trajectory starting at A will move to